Abstract: Wireless communication is growing at a rapid pace in the communication industry. But multipath effect results in incorrect reception of the original transmitted signal. Time-Reversal is a solution to this problem. By concentrating energy in the spatial and temporal domains, Time Reversal (TR) transmission technique provides a low-complexity energy-efficient communication. In this paper, Time-Reversal Division Multiple Access (TRDMA) is used as a wireless channel access method because of its high-resolution spatial focusing effect. An enhanced adaptive version is developed using a QAM modulator for better protection of the signals and a variable energy booster. Simulation results for tests employing time-reversal technique in multipath environment show that desirable properties and satisfying performances are achieved.

Keywords: Time-Reversal, multipath, spatial focusing, TRDMA

1. INTRODUCTION

In a wireless scattering environment, multipath effect makes conventional high-speed communications a challenging task due to the severe Inter-Symbol Interference (ISI). To resolve this problem, multi-carrier modulation (e.g. OFDM) and/or complicated equalization are needed [1]-[4] at the receiver to alleviate the ISI. By concentrating energy in both the spatial and temporal domains, Time-Reversal (TR) transmission technique provides a great potential of low complexity energy-efficient communications [5]. In this paper, time reversal transmission of signals in multipath environment is presented. Also a concept of time-reversal division multiple access (TRDMA) is proposed as a wireless channel access method based on its high-resolution spatial focusing effect. TR structure is used in multi-user downlink systems over multi-path channels, where signals of different users are separated solely by TRDMA [5] [6] [7].

The single-user TR wireless communications consist of two phases: the recording phase and the transmission phase [5]. By utilizing channel reciprocity, TR waves can retrace the incoming paths, focused only at the intended location, as commonly referred to as spatial focusing effect. TR essentially leverages the multi-path channel as a facilitating matched filter for the intended receiver, and concentrates the signal energy in the time domain as well, as commonly referred to as temporal focusing effect.

2. LITERATURE REVIEW

The history of research on TR transmission dates back to early 1990’s. Time-reversal theory was first developed by M.Fink in mid 1990s [5]. However, not much development and interest went beyond the acoustics and ultrasound domains at that time. The concept of TR technology and Time-Reversal Mirror were first used in the optical domain, to obtain high focusing properties. Then it was used in underwater propagation environments. Experiments conducted proved that TRMs could spatially and temporally refocus an incident field [8]. Later it was extended to the concept of iterative time-reversal for waveguide propagation in ocean [9]. When the ocean contains several reflectors, iterative time-
reversal focuses on the target corresponding to the largest eigen value of the TR operator. Since TR makes full use of multi-path propagation and requires no complicated channel processing and equalization, it was tested in wireless radio communication, especially in Ultra-wideband (UWB) systems [10]. Later it was observed that the time-reversal signal transmission is an ideal paradigm for green wireless communications because of its inherent nature to fully harvest energy from the surrounding environment by exploiting the multi-path propagation to re-collect all the signal energy that would have otherwise been lost in most existing communication paradigms. A green wireless technology ensures low energy consumption and low radio pollution to others than the intended user [11].

3. TIME-REVERSAL METHOD IN MULTIPATH ENVIRONMENTS

3.1 Time-Reversal Theory

Previous methods attempted to model the channel and counteract it or introduce redundancy to avoid the effects of multipath propagation [12]. Multipath channel is used as an advantage with TR. Wireless communication mainly deals with electromagnetic waves, which satisfy Helmholtz differential wave equation. Wave equation has the property of reciprocity. Helmholtz wave equation is given by:

$$\nabla^2 G (r, r') + k^2 G (r, r') = -\delta(r - r') \quad (1)$$

k denotes the wave number and function G must conform to boundary conditions of problem. Receiver transmits signal x(t) in forward direction. The effect of the channel is modeled as a filter with impulse response h(t). Received signal is given in equation (2); i represent the array element ranging from 0 to N-1 where N is the number of array elements.

$$r_i(t) = x(t) \otimes h_i(t) \quad (2)$$

Time-reversing this results in r_t(-t). In frequency domain,

$$R_i(f) = X(f)H_i(f) \quad (3)$$

The channel has the same response h (t) in the reverse direction, transmitting r (t) back through the channel results in

$$Y(f) = R_i^*(f)H_i(f) \quad (4)$$

Substituting (3) into (4) yields:

$$Y(f) = X^*(f)H_i^*(f)H_i(f) \quad (5)$$

If the frequency response of the channel is flat over the desired range of frequencies, the output will be:

$$Y(f) = AX^*(f) \quad (6)$$

‘A’ is the scaling factor that takes into account the gain of the filter and the losses associated with propagation. Time-reversing the received signal Y(f) and converting back to time domain results in the same signal that was transmitted, x(t).

3.2 Time-Reversal Division Multiple Access in Multipath Channels

Signal received in a complex environment will have undergone multiple reflections, refractions, and scattering. It consists of the sum of multiple time-delayed and attenuated versions of original signal. When the received signal is time-reversed and transmitted, the different time-delayed components go through the same channel in reverse and converge on the original source location. This convergence occurs in both space and time. The spatial and temporal convergence is the basis of time-reversal focusing. In a signal processing view, time-reversal operation is the convolution of channel with its time-reversed version. TRDMA is used as wireless channel access method based on high-resolution spatial focusing effect [1]. Time-Reversal structure is used in multi-user downlink system over multipath Rayleigh fading channel, where signals of different users are separated by TRDMA.

4. TIME-REVERSAL USING FDTD ALGORITHM

To test the theory behind TR, electromagnetic propagation was modeled using Finite Difference Time Domain algorithm. It involves Maxwell’s Equations and Constitutive Parameters, given by:

$$\nabla \times \vec{E} = -\frac{\partial \vec{D}}{\partial t} \quad (7)$$

$$\nabla \cdot \vec{D} = \rho \quad (8)$$

$$\vec{D} = \varepsilon \vec{E} \quad (9)$$
The total electric field can be defined as:

\[ \vec{E}_{total} = \vec{E}_{incident} + \vec{E}_{scattered} \]  

(13)

Magnetic fields are defined in a similar manner. Assuming free space, \( \sigma = 0 \), and \( \mu = \mu_0 \) and \( \epsilon = \epsilon_0 \). Knowing \( \vec{J} = \epsilon \vec{E} \) and solving these equations, we get:

\[ \frac{\partial \vec{B}}{\partial t} = -\frac{1}{\mu} (\nabla \times \vec{E}) \]  

(10)

\[ \nabla \cdot \vec{B} = 0 \]  

(11)

\[ \vec{B} = \mu \vec{H} \]  

(12)

Now equations are differenced. Finite differencing replaces derivatives with differences [13]:

\[ \frac{\partial \vec{B}}{\partial t} = \lim_{\Delta t \to 0} \frac{f(x,t_2) - f(x,t_1)}{\Delta t} \approx \frac{f(x,t_2) - f(x,t_1)}{\Delta t} \]  

(16)

\[ \frac{\partial \vec{H}}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x_2,t) - f(x_1,t)}{\Delta x} \approx \frac{f(x_2,t) - f(x_1,t)}{\Delta x} \]  

(17)

Applying these to Maxwell’s equations, we get six equations, 2 each in the x, y, and z directions [13]. Two of the equations are:

\[ \frac{\Delta \vec{B}_{\text{scat},n+1/2} - \vec{B}_{\text{scat},n}}{\Delta t} = \frac{1}{\epsilon_0} \left( \frac{\Delta \vec{E}_{\text{scat},n+1/2}}{\Delta y} - \frac{\Delta \vec{E}_{\text{scat},n+1/2}}{\Delta z} \right) \]  

(18)

\[ \frac{\Delta \vec{E}_{\text{scat},n+1/2} - \vec{E}_{\text{scat},n}}{\Delta t} = \frac{1}{\mu_0} \left( \frac{\Delta \vec{B}_{\text{scat},n+1/2}}{\Delta x} - \frac{\Delta \vec{B}_{\text{scat},n+1/2}}{\Delta y} \right) \]  

(19)

To create a leapfrogging phenomenon, electric and magnetic fields have to be staggered. In this manner, the electric fields induce a magnetic field, then the magnetic field induces another electric field and the cycle continues. The time step is determined from Courant condition. In three dimensions, assuming the speed of light is \( c \), the time step must satisfy:

\[ \Delta t = \frac{1}{c \sqrt{\left(\frac{1}{\Delta x}\right)^2 + \left(\frac{1}{\Delta y}\right)^2 + \left(\frac{1}{\Delta z}\right)^2}} \]  

(20)

The simulation chooses one among the three modulation formats: Amplitude Shift Keying (ASK), Phase Shift Keying (PSK) or Frequency Shift Keying (FSK). It also considers the different environments into account: Waveguide, Line Of Sight, No Line Of Sight, Free Space and User defined.

5. SYSTEM MODEL OF TRDMA

5.1 Channel model

We consider a multi-user downlink network for multipath Rayleigh fading channels. The channel impulse response (CIR) of the communication link between the BS and the \( i \)'th user is modeled as:

\[ h_i[k] = \sum_{l=0}^{L-1} h_{i,l}[k-l] \]  

(21)

where \( h_i[k] \) is the k'th tap of the Channel Impulse Response with length L and \( \delta[t] \) is the Dirac delta function. For each link, we assume that \( h_i[k] \)'s are independent circular symmetric complex Gaussian random variables with zero mean and variance:

\[ E[|h_i[k]|^2] = e^{-\frac{k T_s}{\sigma^2}}, \ 0 \leq k \leq L - 1 \]  

(22)

Here \( T_s \) is the sampling period of the system such that \( \frac{1}{T_s} \) equals the system bandwidth B and \( \sigma^2 \) is the root mean square (rms) delay spread of the channel. Each duty cycle consists of the recording phase and the transmission phase, which occupy the proportions of \((1-\eta)\) and \(\eta\) of the cycle period; with \(\eta \in (0,1)\) depending on how fast channels vary over time. CIR’s associated with different users are uncorrelated.

5.2 System model

![Figure 1: SISO TRDMA multi-user downlink system](image)
The system consists mainly of three parts: Base Station, Wireless Channels and Multiple users. The system blocks can best be explained by describing the phases of a time-reversal system. The Single-user TR wireless communications consist of two phases: recording phase and transmission phase.

Recording phase: The N users receive statistically independent messages \(X_1(k), X_2(k), ..., X_N(k)\) from the BS. Time-reversal mirror (TRM) is a device that records and time-reverses (conjugate of complex-valued) received waveform, and modulates the time-reversal waveform with input signal by convolving them in the transmission phase. During the recording phase, the N intended users take turns to transmit an impulse signal to the BS (ideally it can be a Dirac delta-function, but in practice a raised-cosine signal is a good candidate for limited bandwidth). TRMs at the BS record the channel response of each link and store the time-reversed and conjugated version of each channel response for transmission phase.

Transmission phase: Each of the sequences \(\{X_1(k)X_2(k), ..., X_N(k)\}\) is a sequence of information symbols with zero mean and variance \(\theta\). Rate back-off factor \(D\) is introduced as the ratio of sampling rate to baud rate, by performing up-sampling and down-sampling with a factor \(D\) at the BS and receivers as shown in Figure 1. It facilitates simple rate conversion in the analysis of TR.

Sequences are up-sampled by \(D\) at the BS, and the \(i\)-th up-sampled sequence is:

\[
X_i^D[k] = \begin{cases} 
X_i \left[ \frac{k}{D} \right] & \text{if } k \mod D = 0 \\
0 & \text{if } k \mod D \neq 0 
\end{cases}
\] (23)

The up-sampled sequences are fed into the bank of TRMs\(\{g_1, g_2, ..., g_N\}\); the output of \(i\)-th TRM \(g_i\) is the convolution of \(i\)-th up-sampled sequence \(X_i^D[k]\) and TR waveform \(g_i[k]\) with

\[
g_i[k] = \frac{h_i[l-1-k]}{\sqrt{E[\Sigma_{l=0}^{L-1} |h_i[l]|^2]}}
\] (24)

which is the normalized (by the average channel gain) complex conjugate of time-reversed \(h_i[k]\). After that, all the outputs of TRM bank are added together, and the combined signal \(\{S[k]\}\) is transmitted into wireless channels with

\[
S[k] = \sum_{i=1}^{N} (X_i^D[k] \ast g_i)[k]
\] (25)

The signal received at the \(i\)-th user is the convolution of the transmitted signal \(\{S[k]\}\) and the CIR \(\{h[k]\}\) plus an additive white Gaussian noise, given by:

\[
Y_i[D][k] = \sum_{j=1}^{N} \left( X_j^D[k] \ast g_j \ast h_i \right)[k] + n_i[k]
\] (26)

Due to temporal focusing effect, signal energy is concentrated in a single time sample. The \(i\)-th user performs a one-tap gain adjustment to the received signal to recover it and then down-samples it with same factor \(D\), ending up with \(Y_i[k]\).

\[
Y_i[k] = a_i \sum_{j=1}^{N} \left( (h_i \ast g_j)[D] \ast X_j \right)[k-l] + a i n_i[k]
\] (27)

\[
(h_i \ast g_j)[k] = \sum_{l=0}^{\infty} h_i[l]g_j[k-l] = 0\text{for }l=0\text{to }L-1,
\] (28)

with \(k=0, 1, 2, ..., 2L-2\) and \(n_i[k] = \bar{n}_i[Dk]\), which is AWGN with zero mean and variance \(\sigma^2\).

A MISO case is considered where the transmitting BS is equipped with \(M_r\) antennas along with multiple single-antenna users. The diagram for the MISO TRDMA is shown in Figure 2.

Let \(h_i^{(m)}[k]\) denote the \(k\)-th tap of the CIR for the communication link between user \(i\) and the \(m\)-th antenna of the BS. It is a circular symmetric complex Gaussian random variable with zero mean and variance:

\[
E\left[|h_i^{(m)}[k]|^2\right] = e^{-\frac{kr_s}{\sigma}}
\] (29)

For MISO, each antenna at BS is similar to a single-antenna BS in the SISO. TR waveform \(\{g_i^{(m)}[k]\}\) is
the normalized complex conjugate of time-reversed\( h_i^{(m)}[k] \), i.e.
\[
g_i^{(m)}[k] = \frac{h_i^{(m)*}[l-1-k]}{\sqrt{E[M_r \sum_{m=0}^{M_r-1} |h_i^{(m)}[l]|^2]}} \tag{30}
\]

So the average total transmit power at the BS is
\[
P = \frac{N \times d}{D} \tag{31}
\]
which does not depend on the number of transmit antennas, \( M_r \). The resulting received signal at user \( i \) is represented as:
\[
Y_i[k] = \sum_{j=1}^{N} \sum_{m=1}^{M_r} \sum_{l=0}^{2L-2} (h_i^{(m)} \ast g_j^{(m)})[Dl]X_j[k-l] + n[k] \tag{32}
\]
where \( n[k] \) is AWGN with zero mean and variance \( \sigma^2 \). An improvement in the model is done by performing a QAM modulation to provide better security to the TRDMA system.

6. SIMULATION RESULTS FOR TR IN MULTIPATH SCENARIO

To have a clear understanding of time-reversal in multipath scenario, electromagnetic propagation is modeled using Finite Difference Time Domain (FDTD) algorithm. This method considers antenna arrays into account. After each time step of the FDTD algorithm, electric field is plotted. The result is a slideshow that depicts the wave propagation from the receiver to the array. Figure 3 shows an instant of forward wave propagation.

When the entire signal is transmitted, the signal received at the array is plotted. Figure 4 is the plot after forward propagation. The signal is then time reversed. It is sent back to the original transmitter. The signal propagates back through the same environment through which forward propagation occurred. Figure 5 is an instant of backward wave propagation.

6. SIMULATION RESULTS FOR TR IN MULTIPATH SCENARIO

To have a clear understanding of time-reversal in multipath scenario, electromagnetic propagation is modeled using Finite Difference Time Domain (FDTD) algorithm. This method considers antenna arrays into account. After each time step of the FDTD algorithm, electric field is plotted. The result is a slideshow that depicts the wave propagation from the receiver to the array. Figure 3 shows an instant of forward wave propagation.

When the entire signal is transmitted, the signal received at the array is plotted. Figure 4 is the plot after forward propagation. The signal is then time reversed. It is sent back to the original transmitter. The signal propagates back through the same environment through which forward propagation occurred. Figure 5 is an instant of backward wave propagation.

Figure 4: Received multipath signal at the array

Figure 5: Wave propagation from array to receiver

It is seen that the signal focuses in space and time and reaches only at the intended receiver. These are the peculiar properties of Time-Reversal. Now the signal is time-reversed again and the received signal is shown in Figure 6.

Figure 6: Received signal
In order to ensure that simulation will run for a long enough period of time in the reverse direction, the time reversed signal was zero padded. As seen, the received signal does not have multipath components. So TR, with its spatial focusing, is an efficient technique for multipath environment. All the processes taking place in the time reversal simulation as slideshow is saved as a movie in AVI format in the working directory of MATLAB.

7. PERFORMANCE EVALUATION OF TRDMA

TRDMA scheme uses TR structure in the system model of multi-user downlink system. Performance evaluation is done from SINR and a performance booster is used for signals in low SNR regime. A QAM modulator is added to give better security. A modified received signal to noise ratio is defined:

$$\rho = \frac{P}{\sigma^2} E[\sum_{l=0}^{L-1} |Y_i[l]|^2] = \frac{P}{\sigma^2} \frac{LT}{{\eta}^2}$$

(33)

Average effective SINR:

As SISO is a special case with $M_r = 1$, we analyze the general MISO case with $M_r$ as parameter. The $i$-th receiver is designed to estimate $X_i[k - \frac{L-1}{D}]$ based on observation of $Y_i[k]$. Remaining components of $Y_i$ is categorized as inter-symbol interference, inter-user interference and noise.

$$Y_i[k] = a_i \sum_{m=1}^{M_T} (h_i^{(m)} * g_i^{(m)})[L - 1]X_i[k - \frac{L-1}{D}]$$

$$+ \sum_{l=0}^{2L-2} \frac{D}{2D} \sum_{i=0}^{M_T} \sum_{l=0}^{M_T} (h_i^{(m)} * g_i^{(m)})[D]X_i[k - l]$$

$$+ a_i \sum_{j=1}^{N} \sum_{l=0}^{M_T} \sum_{m=1}^{M_T} (h_j^{(m)} * g_j^{(m)})[D]X_j[k - l]$$

$$+ n[k]$$

(34)

The first term is signal, second is ISI; third is IUI and fourth term represents noise. Since tap gain $a_i$ does not affect the effective SINR, we consider $a_i = 1$. The signal power $P_s(i)$ is:

$$P_s(i) = E \left[ \sum_{m=1}^{M_T} (h_i^{(m)} * g_i^{(m)})[L - 1]X_i[k - \frac{L-1}{D}]^2 \right]$$

$$= \theta \sum_{m=1}^{M_T} (h_i^{(m)} * g_i^{(m)})[L - 1]^2$$

(35)

where $E[.]$ represents the expectation over $X$. Accordingly, the powers associated with ISI and IUI can be derived as:

$$P_{ISI}(i) = \theta \sum_{l=0}^{L-2} \sum_{j=0}^{M_T} \sum_{m=1}^{M_T} (h_i^{(m)} * g_j^{(m)})[D]I[l]^2$$

(36)

$$P_{UUI}(i) = \theta \sum_{l=0}^{L-2} \sum_{j=1}^{N} \sum_{m=1}^{M_T} (h_i^{(m)} * g_j^{(m)})[D]I[l]^2$$

(37)

SINR is a crucial performance metric used to measure the extent to which a signal is corrupted. It is especially the case for a media-access scheme, where interference management is one of the main design objectives. The average effective SINR at user $i$ $SINR_{avg}(i)$ is defined as the ratio of the average signal power to the average noise-and-interference power.

$$SINR_{avg}(i) = \frac{E[P_{sig}(i)]}{E[P_{ISI}(i)] + E[P_{IUI}(i)] + \sigma^2}$$

(38)

We choose $L = 257$ and $\sigma_T = 128T_S$ from a typical range, and evaluate the average effective SINR versus $\rho$ in terms of $N$ (number of users), $M_r$ (number of antennas) and $D$ (rate back-off factor).

Figure 7: The impact of rate back-off factor when $N=5$, $M_r=4$
Figure 7 is plotted with $M_r = 4$, $N=5$. In the high SNR area where interference powers dominate noise power, nearly a 3dB gain in effective SINR is seen when D is doubled. In low SNR region, the SINR was found to be low. Figure 8 shows the symbol error rate for a TRDMA system. In the low SNR regime, the error rate is more. But in the high SNR region, the error rate decreases drastically.

TRDMA is made into an adaptive system by defining a threshold in the low SNR region. To reduce symbol interference, interpolation is done. But delays due to interpolation are undesirable. So signal amplification is done with an energy booster to improve the performance in low SNR region. Receiver compares the energy with the threshold and decides whether amplification has to be done. When signal energy is below the threshold level, energy boosting is done. Such amplification is done only if the performance is low even after up-sampling. Output is fed back allowing response to changes, making it an adaptive system. Figure 9 shows the plot of average effective SINR of a TRDMA system with energy booster. The bottom plot shows the signal below threshold, next is the signal at threshold and the first two plots are the amplified signals. In the high SNR region, noise is not so predominant compared to interference.

8. CONCLUSIONS AND FUTURE WORK

The Time-Reversal transmission technique for wireless communications in multipath environments was discussed in the paper. TR technique makes multipath an advantage for transmission. It helps in enhancing the performance of a wireless communication system in multipath scenario. The two most important properties of time-reversal; spatial focusing and temporal compression, was presented in the paper. The focusing ability of the TR waves was tested and positive results were obtained. Also a TRDMA scheme for multipath downlink network was investigated. Performance evaluation of TRDMA was done in terms of average effective SINR and symbol error rate. An adaptive TRDMA scheme was developed for low SNR region. TRDMA can be a promising technique in the future energy-efficient low-complexity broadband wireless communications. In the electromagnetic simulation using FDTD algorithm, the shape of the array considered was linear. In future, other array shapes can be tested using this simulation to determine whether different shapes can improve the focusing ability of the array. Also this can be used to transmit power to objects without knowing the exact location of the object since time-reversed waveforms follow the same path in the backward propagation. Tests could also be performed to determine how close the users can get to one another in the TRDMA model before the interference between the two becomes an important factor. There is a lot of room for growth in this
field, and simulations can be a very useful tool in fleshing out the nuances.

REFERENCES